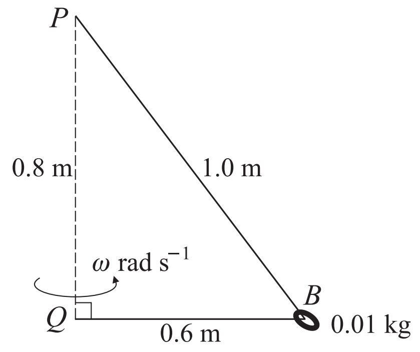
OCR Maths M2

Topic Questions from Papers

Circular Motion

1



One end of a light inextensible string of length 1.6 m is attached to a point P. The other end is attached to the point Q, vertically below P, where $PQ = 0.8$ m. A small smooth bead B, of mass 0.01 kg, is threaded on the string and moves in a horizontal circle, with centre Q and radius 0.6 m. QB rotates with constant angular speed ω rad s^{-1} (see diagram).

(i) Show that the tension in the string is 0.1225 N. [3]

(ii) Find ω. [3]

(iii) Calculate the kinetic energy of the bead. [2]

(Q3, June 2005)

2

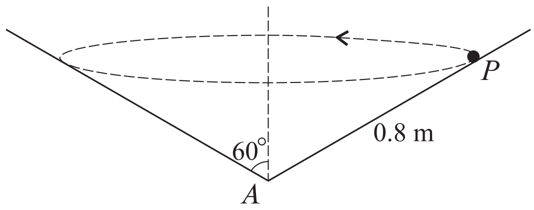


Fig. 1

A particle P of mass 0.1 kg is moving with constant angular speed ω rad s^{-1} in a horizontal circle on the smooth inner surface of a cone which is fixed with its axis vertical and its vertex A at its lowest point. The semi-vertical angle of the cone is 60° and the distance AP is 0.8 m (see Fig. 1).

(i) Calculate the magnitude of the force exerted by the cone on the particle. [3]

(ii) Calculate ω. [4]

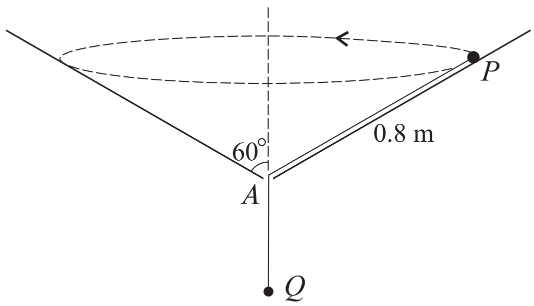


Fig. 2

The particle P is now attached to one end of a light inextensible string which passes through a small smooth hole at A. The lower end of the string is attached to a particle Q of mass 0.2 kg. Q is in equilibrium with the string taut and $AP = 0.8$ m. P moves in a horizontal circle with constant speed v m s^{-1} (see Fig. 2).

(iii) State the tension in the string. [1]

(iv) Find v. *(Q8, Jan 2006)* [6]

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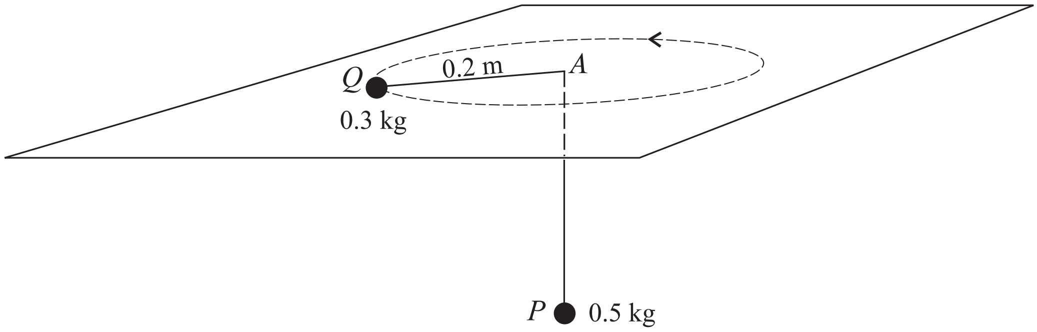


Fig. 1

A light inextensible string of length 1 m passes through a small smooth hole A in a fixed smooth horizontal plane. One end of the string is attached to a particle P, of mass 0.5 kg, which hangs in equilibrium below the plane. The other end of the string is attached to a particle Q, of mass 0.3 kg, which rotates with constant angular speed in a circle of radius 0.2 m on the surface of the plane (see Fig. 1).

(i) Calculate the tension in the string and hence find the angular speed of Q. [4]

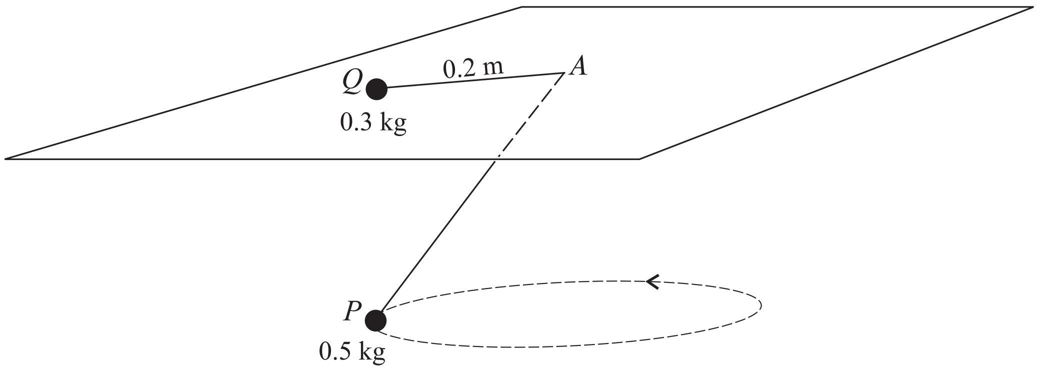


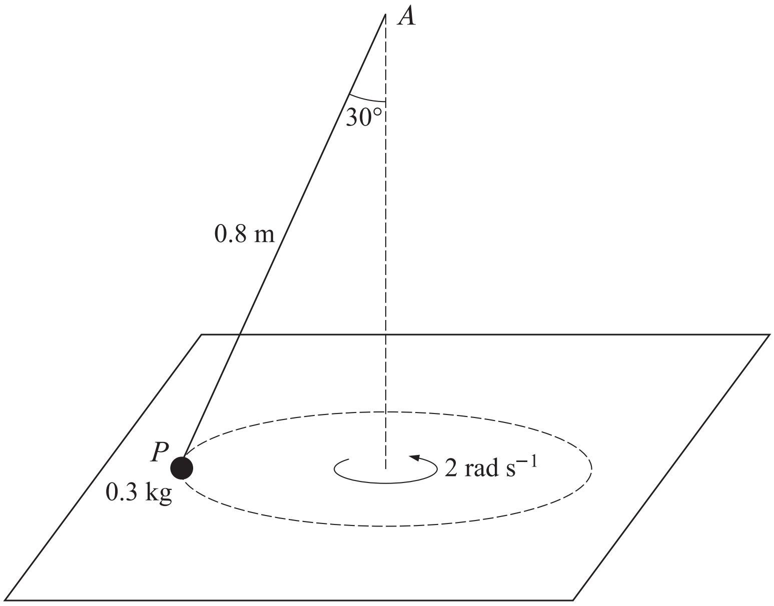
Fig. 2

The particle Q on the plane is now fixed to a point 0.2 m from the hole at A and the particle P rotates in a horizontal circle of radius 0.2 m (see Fig. 2).

(ii) Calculate the tension in the string. [4]

(iii) Calculate the speed of P. [3]

(Q6, June 2006)



One end of a light inextensible string of length 0.8 m is attached to a fixed point A which lies above a smooth horizontal table. The other end of the string is attached to a particle P, of mass 0.3 kg, which moves in a horizontal circle on the table with constant angular speed 2 rad s^{-1}. AP makes an angle of 30° with the vertical (see diagram).

(i) Calculate the tension in the string. [4]

(ii) Calculate the normal contact force between the particle and the table. [3]

The particle now moves with constant speed v m s^{-1} and is on the point of leaving the surface of the table.

(iii) Calculate v. [6]

(Q7, Jan 2007)

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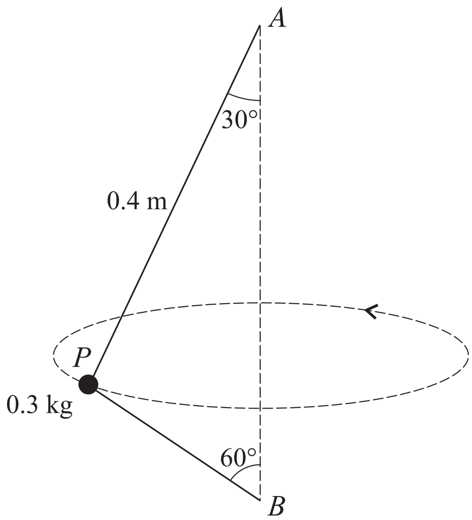

A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point A and the other end of the shorter string is attached to a fixed point B, which is vertically below A. AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string AP is 5 N.

Calculate

(i) the tension in the string PB, [3]

(ii) the angular speed of P, *(Q6, June 2007)* [3]

6 **(i)**

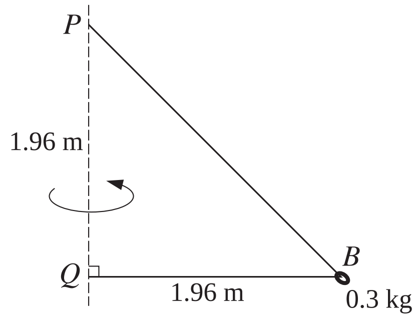

Fig. 1

One end of a light inextensible string is attached to a point P. The other end is attached to a point Q, 1.96 m vertically below P. A small smooth bead B, of mass 0.3 kg, is threaded on the string and moves in a horizontal circle with centre Q and radius 1.96 m. B rotates about Q with constant angular speed ω rad s^{-1} (see Fig. 1).

(a) Show that the tension in the string is 4.16 N, correct to 3 significant figures. [2]

(b) Calculate ω. [3]

(ii)

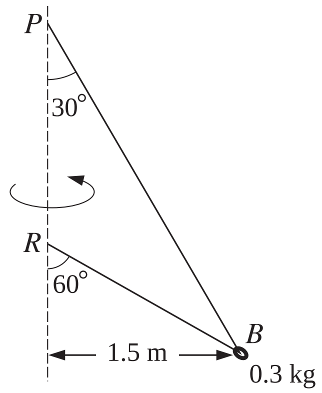

Fig. 2

The lower part of the string is now attached to a point R, vertically below P. PB makes an angle 30° with the vertical and RB makes an angle 60° with the vertical. The bead B now moves in a horizontal circle of radius 1.5 m with constant speed v m s^{-1} (see Fig. 2).

(a) Calculate the tension in the string. [3]

(b) Calculate v. [3]

(Q6, Jan 2008)

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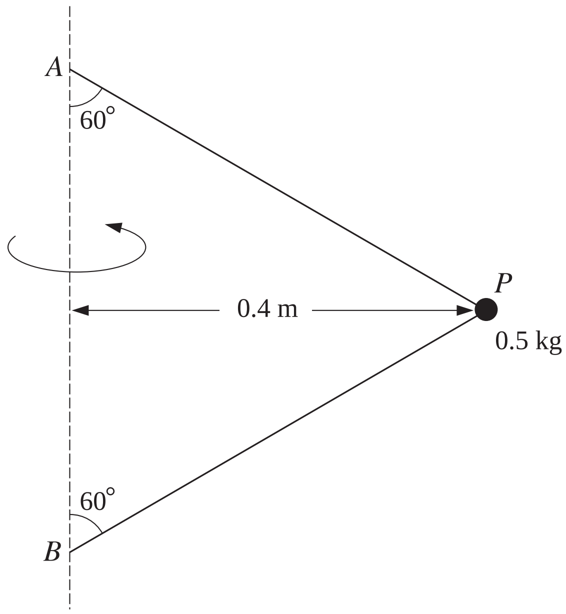

A particle P of mass 0.5 kg is attached to points A and B on a fixed vertical axis by two light inextensible strings of equal length. Both strings are taut and each is inclined at 60° to the vertical (see diagram). The particle moves with constant speed 3 m s^{-1} in a horizontal circle of radius 0.4 m.

(i) Calculate the tensions in the two strings. [7]

The particle now moves with constant angular speed ω rad s^{-1} and the string BP is on the point of becoming slack.

(ii) Calculate ω. [5]

(Q6, June 2008)

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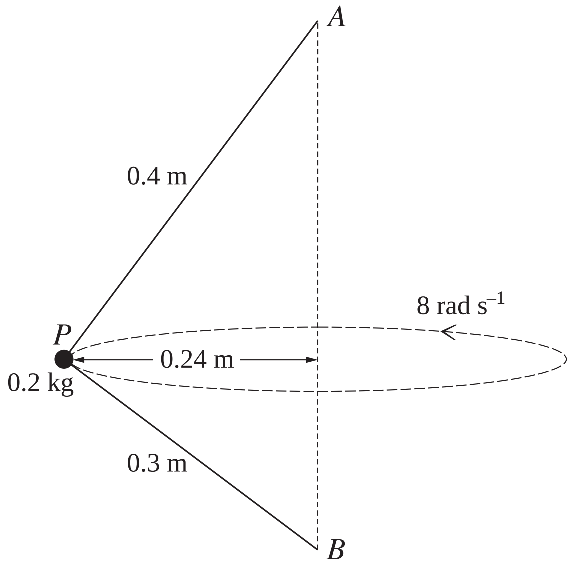

A particle P of mass 0.2 kg is attached to one end of each of two light inextensible strings, one of length 0.4 m and one of length 0.3 m. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The particle moves in a horizontal circle of radius 0.24 m at a constant angular speed of 8 rad s^{-1} (see diagram). Both strings are taut, the tension in AP is S N and the tension in BP is T N.

(i) By resolving vertically, show that $4S = 3T + 9.8$. **[4]**

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(ii) Find another equation connecting S and T and hence calculate the tensions, correct to 1 decimal place. **[8]**

(Q5, Jan 2009)

9 A light inextensible string of length 0.6 m has one end fixed to a point A on a smooth horizontal plane. The other end of the string is attached to a particle B, of mass 0.4 kg, which rotates about A with constant angular speed $2\,\text{rad}\,\text{s}^{-1}$ on the surface of the plane.

(i) Calculate the tension in the string. **[2]**

A particle P of mass 0.1 kg is attached to the mid-point of the string. The line APB is straight and rotation continues at $2\,\text{rad}\,\text{s}^{-1}$.

(ii) Calculate the tension in the section of the string AP. **[4]**

(iii) Calculate the total kinetic energy of the system. **[5]**

(Q4, June 2009)

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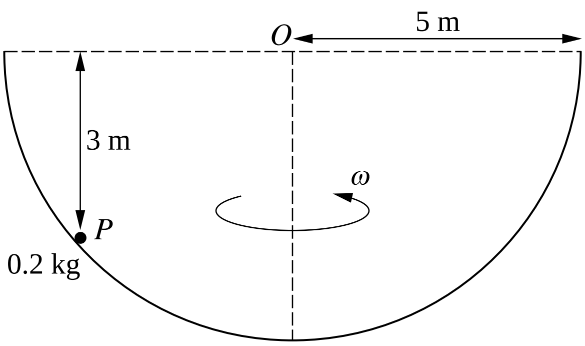

Fig. 1

A particle P of mass 0.2 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre O and radius 5 m. P moves with constant angular speed ω in a horizontal circle at a vertical distance of 3 m below the level of O (see Fig. 1).

(i) Calculate the magnitude of the force exerted by the hemisphere on P. **[3]**

(ii) Calculate ω. **[4]**

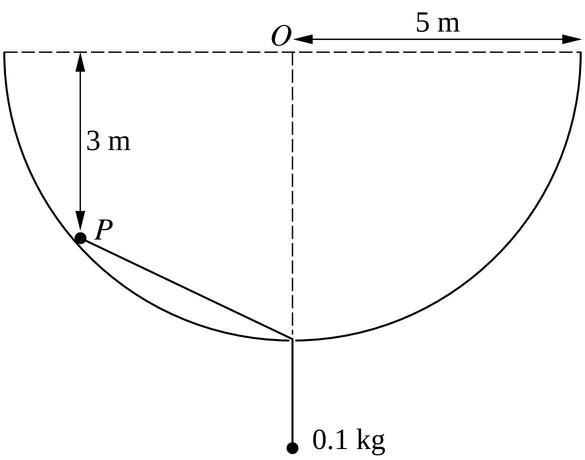

Fig. 2

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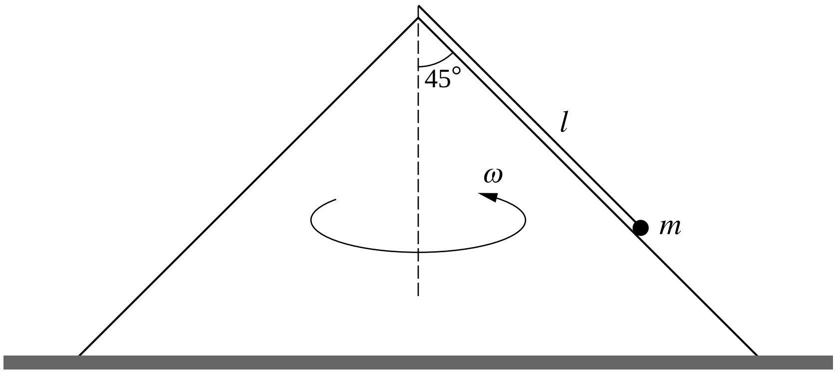
A light inextensible string is now attached to P. The string passes through a small smooth hole at the lowest point of the hemisphere and a particle of mass 0.1 kg hangs in equilibrium at the end of the string. P moves in the same horizontal circle as before (see Fig. 2).

(iii) Calculate the new angular speed of P. **[8]**

(Q7, Jan 2010)

11



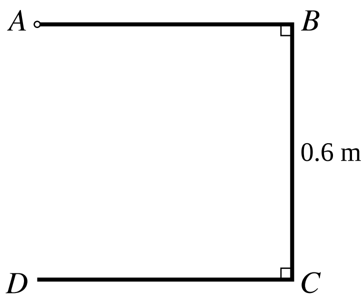
One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45°. The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R.

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. **[6]**

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. **[4]**

(Q5, June 2010)

12



A uniform square frame $ABCD$ has sides of length 0.6 m. The side AD is removed from the frame, and the open frame $ABCD$ is attached at A to a fixed point (see diagram).

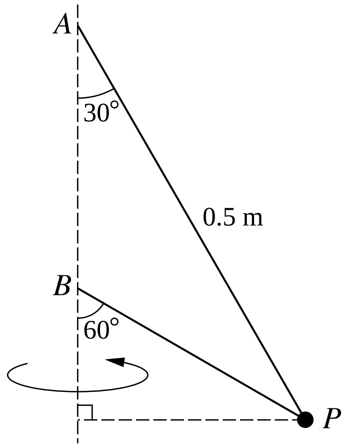
(i) Calculate the distance of the centre of mass of the open frame from A. **[5]**

The open frame rotates about A in the plane $ABCD$ with angular speed 3 rad s^{-1}.

(ii) Calculate the speed of the centre of mass of the open frame. **[2]**

(Q1, Jan 2011)

13



A particle P of mass 0.4 kg is attached to one end of each of two light inextensible strings which are both taut. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The string AP makes an angle of 30° with the vertical and is 0.5 m long. The string BP makes an angle of 60° with the vertical. P moves with constant angular speed in a horizontal circle with centre vertically below B (see diagram). The tension in the string AP is twice the tension in the string BP. Calculate

(i) the tension in each string, **[4]**

(ii) the angular speed of P. **[4]**

(Q3, Jan 2011)

14

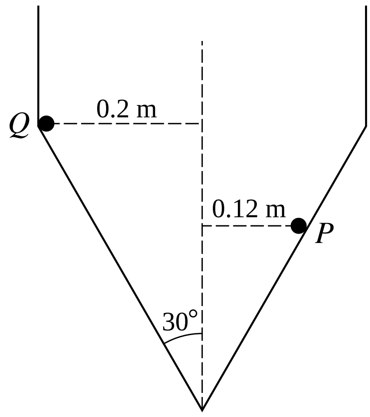


Fig. 1

A container is constructed from a hollow cylindrical shell and a hollow cone which are joined along their circumferences. The cylindrical shell has radius 0.2 m, and the cone has semi-vertical angle 30°. Two identical small spheres P and Q move independently in horizontal circles on the smooth inner surface of the container (see Fig. 1). Each sphere has mass 0.3 kg.

(i) P moves in a circle of radius 0.12 m and is in contact with only the conical part of the container. Calculate the angular speed of P. **[5]**

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(ii)

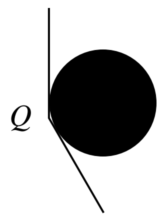


Fig. 2

Q moves with speed $2.1\,\text{m s}^{-1}$ and is in contact with both the cylindrical and conical surfaces of the container (see Fig. 2). Calculate the magnitude of the force which the cylindrical shell exerts on the sphere. **[4]**

(iii) Calculate the difference between the mechanical energy of P and of Q. **[5]**

(Q6, June 2011)

15 A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is fixed at a point A which is 0.6 m above a smooth horizontal table. P moves on the table in a circular path whose centre O is vertically below A.

(i) Given that the angular speed of P is $2.5\,\text{rad s}^{-1}$, find

(a) the tension in the string, **[4]**

(b) the normal reaction between the particle and the table. **[3]**

(ii) Find the greatest possible speed of P, given that the particle remains in contact with the table. **[5]**

(Q4, Jan 2012)

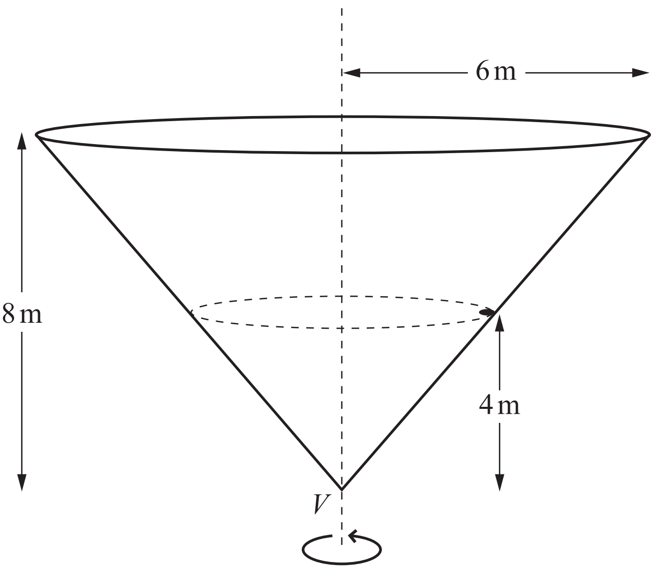
16 A particle P, of mass 2 kg, is attached to fixed points A and B by light inextensible strings, each of length 2 m. A and B are 3.2 m apart with A vertically above B. The particle P moves in a horizontal circle with centre at the mid-point of AB.

(i) Find the tension in each string when the angular speed of P is $4\,\text{rad s}^{-1}$. **[7]**

(ii) Find the least possible speed of P. **[6]**

(Q5, June 2012)

17



A conical shell has radius 6 m and height 8 m. The shell, with its vertex V downwards, is rotating about its vertical axis. A particle, of mass 0.4 kg, is in contact with the rough inner surface of the shell. The particle is 4 m above the level of V (see diagram). The particle and shell rotate with the same constant angular speed. The coefficient of friction between the particle and the shell is μ.

(i) The frictional force on the particle is F N, and the normal force of the shell on the particle is R N. It is given that the speed of the particle is 4.5 m s^{-1}, which is the smallest possible speed for the particle not to slip.

(a) By resolving vertically, show that $4F + 3R = 19.6$. **[2]**

(b) By finding another equation connecting F and R, find the values of F and R and show that $\mu = 0.336$, correct to 3 significant figures. **[6]**

(ii) Find the largest possible angular speed of the shell for which the particle does not slip. **[6]**

(Q8, Jan 2013)

18 A vertical hollow cylinder of radius 0.4 m is rotating about its axis. A particle P is in contact with the rough inner surface of the cylinder. The cylinder and P rotate with the same constant angular speed. The coefficient of friction between P and the cylinder is μ.

(i) Given that the angular speed of the cylinder is 7 rad s^{-1} and P is on the point of moving downwards, find the value of μ. **[5]**

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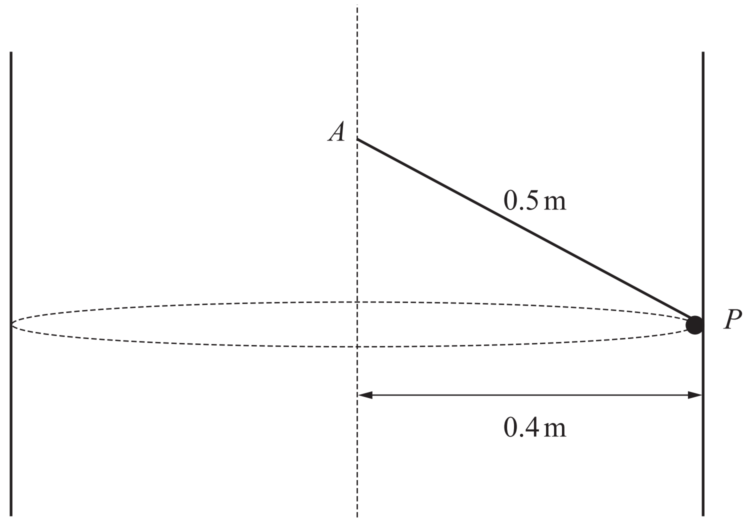
The particle is now attached to one end of a light inextensible string of length 0.5 m. The other end is fixed to a point A on the axis of the cylinder (see diagram).



(ii) Find the angular speed for which the contact force between P and the cylinder becomes zero. **[5]**

(Q5, June 2013)